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Determination Of Confluence Points Of Parked Vehicle Demand By Applying Polynomial And Interpolation Functions

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Abstract

In this research work the implementation of a mathematical model to a road is carried out where the mathematical procedure is intended to be applied in the road of the urban sector of a municipality, mathematically modeling its behavior in order to find the strategic location of the parking lots and their vehicular capacities. Initially, according to a study carried out, it is obtained from some primary interpolation functions that are determined in the x direction from streets 2 to 7, the interpolation polynomials shown in the matrices are calculated, in such a way that the interpolation functions of streets 2 to 7 are obtained. Once these primary functions are obtained, the secondary interpolation functions of these streets are calculated, where the interpolation functions of the coefficients a, b, c and d are obtained. Finally, the surface equation is performed, where the number of vehicles parked in R3 is obtained, as well as the system of equations for the solution of the confluence points of the demand, thus obtaining the points within the study area.

Keywords: model; demand; functions; interpolation; polynomials.

1. Introduction

The importance of the study lies in the need to give the problem of saturation and reduction of mobility an optimal solution that professionally guarantees positive results, to solve the crisis of vehicular demand during peak hours of tourism (Oña López, J; Oña López, R. (2018).

In the texts of numerical analysis, the Lagrange interpolation polynomial (Khan Academy (2019), corresponding to n + 1 given values, is defined as that polynomial function of degree at most n that takes on the n + 1 different numerical values {x, x,..., or 1 x), the n + 1 given values ú, y,..., y). The explanation

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that is usually given to obtain such a polynomial naturally depends on the author's own theoretical orientation. In some texts consulted, an interpolating polynomial is proposed as a formal expression f(x)=a 0 +ax1+ax2 +...+axn, in which the coefficients a 0,..., a are determined by the conditions of the problem, that is, they are asked to satisfy the following system of equations:

Figure 1. System of equations

$$
\begin{aligned}
a_0 + a_1 x_0 + a_2 x_0^2 + \ldots + a_n x_0^n &= y_0 \\
a_0 + a_1 x_1 + a_2 x_1^2 + \ldots + a_n x_1^n &= y_1 \\
&\ldots \\
a_0 + a_1 x_n + a_2 x_n^2 + \ldots + a_n x_n^n &= y_n
\end{aligned}
$$

Source. Enseñanza de la Matemática (2002).

2. Method

This research is carried out according to the following steps and theoretical bases:

- The study area is determined for streets 2 to 7 in the municipality of Chinácota.
- Application of polynomial functions of one way $i\ fi(x) = C_G * X^G + C_{G-1} * X^{G-1} + C_{G-2} * X^{G-2} \ldots C_{G-G} * X^{G-G}$ (Cárdenas - Gutiérrez J.A, NJ Cely-Calixto, Carlos Acevedo Peñaloza, 2022).
- Application of secondary interpolation functions to determine coefficients a,b,c and d (Ayres Frank, Jr., 1989).
- Application of surface equation for parked vehicles in R3.
- Demand confluence points within the study area.

3. Results and discussion

3.1. Application of the model to the sample

3.1.1. Primary interpolation functions

Based on the theoretical basis of this paper (Chapra Steven, C., Canale Raymond, P., (2007), the primary polynomial interpolation functions (Prieto G., (2019), determined in the X direction (lanes 2 to 7), are obtained as follows:

- *Street 2*

Based on the data record obtained in the field, we obtain (Larson, Roland E., Edwards, B., (2010):

Table 1. Tabulation of points (x,VP) equivalent to vehicles parked by race.

Road	2	3	4	5
parked vehicles (VP)	1,0	4,0	5,0	2,5

Transposing the data, the vector <x, VP> (Morales Ramón, J., (2013) is obtained:

Source: Own (2022)

Table 2. Vector <x, VP>.

x	VP
2	1
3	4
4	5
5	2,5

Source: Own (2022)

Following the procedure of the proposed model, the interpolation polynomial is calculated (Reyes de la Cruz, R., (2012), as shown.

Equation 1. Polynomial interpolation matrix.

a	b	c	d	
8	4	2	1	1
27	9	3	2	4
64	16	4	3	5
125	25	5	4	2,5

Source: Own (2022)

Solution matrix or vector of coefficients of the interpolation function (Cerino, F. 2020) for street 2.

a	-0,25
b	1,25
c	-3,5
d	5

Finally, we arrive at the interpolation function.

Equation 2. Polynomial interpolation function of street 2.

$$VP = -0.25x3 + 1.25x2 - 3.5x + 5.$$

– *Street 3*

Based on the data record obtained in the field, it is obtained:

Table 4. Tabulation of points (x,VP) equivalent to parked vehicles per run.

Road	2	3	4	5
parked vehicles (VP)	5,00	13,50	10,50	4,50

Source: Own (2022)

Table 5. Vector <x,VP>.

x	VP
2	5
3	13,5
4	10,5
5	4,5

Following the procedure of the proposed model, the interpolation polynomial is calculated as shown.

Equation 2. Polynomial interpolation matrix.

a	b	c	d	
8	4	2	1	5
27	9	3	2	13,5
64	16	4	3	10,5
125	25	5	4	4,5

Source: Own (2022)

Table 6. Solution matrix or vector of coefficients of the interpolation function (Peña Ferrández, J. M., & Carnicer Álvarez, J. M. 2018) for street 3.

a	1,4166667
b	-18,5
c	-6,416667
d	80,5

Equation 3. Polynomial interpolation function of lane 3.

$$VP = 1.416x3 - 18.5x2 - 6.4167x + 80.5.$$

– *Street 4*

Based on the data record obtained in the field, it is obtained:

Table 7. Tabulation of points (x,VP) equivalent to parked vehicles per run.

Road	2	3	4	5
parked vehicles (VP)	5,0	17,0	13,5	3,0

Transposing the data, the vector <x, VP> is obtained:

Table 8. Vector <x, VP>. Source: Own (2020)

x	VP
2	5
3	17
4	13,5
5	3

Following the procedure of the model, the interpolation polynomial is calculated as shown.

Equation 4. Polynomial interpolation matrix.

a	b	c	d	
8	4	2	1	5
27	9	3	2	17
64	16	4	3	13,5
125	25	5	4	3

Source: Own (2022)

Solution matrix or vector of coefficients of the interpolation function for street 4.

a	1,4166667
b	-20,5
c	-11,91667
d	99,5

Equation 5. Polynomial interpolation function of lane 4.

$$VP = 1.4167x3 - 20.5x2 - 11.9167x + 99.5.$$

– *Street 5*

Based on the data record obtained in the field, it is obtained:

Table 10. Tabulation of points (x,VP) equivalent to parked vehicles per run.

Road	2	3	4	5
parked vehicles (VP)	6,00	11,50	17,00	13,00

Source: Own (2020)

Transposing the data, the vector <x, VP> is obtained (Morales Ramón, J., (2013):

Table 11. Vector <x, VP>. Source: Own (2020)

x	VP
2	6
3	11,5
4	17
5	13

Following the procedure of the posed model, the interpolation polynomial (Lazarus, C. C. (2018) is calculated as shown.

Equation 6. Polynomial interpolation matrix).

a b c d

8	4	2	1	6
27	9	3	2	11,5
64	16	4	3	17
125	25	5	4	13

Source: Own 2022.

Solution matrix or coefficient vector of the interpolation function (Reyes de la Cruz, R., (2012) for street 5.

a	-1,583333
b	14,25
c	-2,666667
d	-33

Equation 7. Polynomial interpolation function of lane 5.

VP = -1, 5833x3 + 14, 25x2 - 2, 6667x - 33.

– *Street 6*

Based on the data record obtained in the field, it is obtained:

Table 13 Tabulation of points (x,VP) equivalent to parked vehicles per run.

Road	2	3	4	5
parked vehicles (VP)	4,00	4,50	7,50	4,00

Source: Own (2022)

Transposing the data, the vector <x,VP> is obtained:

Table 14. vector <x,VP>. Source: Own (2022)

x	VP
	4
3	4,5
4	7,5
5	4

Following the procedure of the proposed model, the interpolation polynomial is calculated as shown.

Equation 8. Polynomial interpolation matrix.

a	b	c	d	
8	4	2	1	4
27	9	3	2	4,5
64	16	4	3	7,5
125	25	5	4	4

Source: Own (2020)

Table 15. Solution matrix or vector of coefficients of the interpolation function for street 6.

a	-1,5
b	14,75
c	1,75
d	-46,5

Equation 9. Polynomial interpolation function of lane 6.

$$VP = -1.5x3 + 14.75x2 + 1.75x - 46.5.$$

– *Street 7*

Based on the data record obtained in the field, it is obtained:

Table 16. Tabulation of points (x, VP) equivalent to parked vehicles per race.

Road	2	3	4	5
parked vehicles (VP)	2,0	3,0	3,0	3,0

Source: Own (2022)

Transposing the data, the vector <x, VP> is obtained:

Table 17. Vector <x, VP>. Source: Own (2022)

x	VP
	2
3	3
4	3
5	3

Following the procedure of the proposed model, the interpolation polynomial is calculated as follows

Equation 10. Polynomial interpolation matrix.

a	b	c	d	
8	4	2	1	2
27	9	3	2	3
64	16	4	3	3
125	25	5	4	3

Source: Own (2022)

Solution matrix or coefficient vector of the interpolation function for street 7.

a	0,1666667
b	-2
c	0,8333333
d	7

Equation 11. Polynomial interpolation function of lane 7.

$$VP = 0.166x3 - 2x2 + 0.8333x + 7.$$

Thus, the interpolation functions for streets 2 to 7 are:

Street 2: $VP = 1.4167x3 - 20.5x2 - 11.9167x + 99.5$

Street 3: VP = 1.416x3 - 18.5x2 - 6.4167x + 80.5

Street 4: VP = 1.4167x3 - 20.5x2 - 11.9167x + 99.5

Street 5: VP = -1.5833x3 + 14.25x2 - 2.6667x – 33

Street 6: VP = -1.5x3 + 14.75x2 + 1.75x - 46.5

Street 7: VP = 0.166x3 - 2x2 + 0.8333x + 7.

3.1.2. Secondary interpolation functions

Based on what is consigned in the equations of (Cárdenas - Gutiérrez J.A, NJ Cely-Calixto, Carlos Acevedo Peñaloza, (2022), the secondary polynomial interpolation functions, determined in the Y direction (runs 2 to 5), are obtained as follows.

The respective coefficients a, b, c, d, for the different modeled streets from (2 to 7), are as follows:

Table 19. Tabulation of coefficients of the interpolation function by street.

y	7	6	5	4	3	2
a	0,1666667	-1,5	1,4166667	1,4166667	1,4166667	-0,25
b	-2	14,75	-20,5	-18,5	-18,5	1,25
c	0,8333333	1,75	-2,666667	-11,91667	-6,416667	-3,5
d	7	-46,5	-33	99,5	80,5	5

Source: Own (2022)

– *Coefficient a*

Based on the coefficients presented in Table 19, the following are obtained:

Table 20. tabulation of points (y, a).

y	7	6	5	4	3	2
a	0,1666667	-1,5	1,4166667	1,4166667	1,4166667	-0,25

Source: Own (2022)

Table 21. Vector <y, a>.

y	a
7	0,1666667
6	-1,5
5	-1,583333
4	1,4166667
3	1,4166667
2	-0,25

Source: Own (2020)

Following the procedure of the proposed model, the interpolation polynomial of the coefficient a is calculated as shown:

Equation 12. Polynomial interpolation matrix of coefficient a.

A	B	C	D	E	F	
16807	2401	343	49	7	1	0,1666667
7776	1296	216	36	6	2	-1,5
3125	625	125	25	5	3	-1,583333
1024	256	64	16	4	4	1,4166667
243	81	27	9	3	5	1,4166667
32	16	8	4	2	6	-0,25

Source: Own (2020)

Table 22. Solution matrix or vector of coefficients of the interpolation function for coefficient a.

A	-0,125
B	2,8090278
C	-23,92361
D	95,607639
E	-162,5556
F	15,479167

Source: Own (2022)

Equation 13. Interpolation function of this coefficient.

$$a = -0.125y5 + 2.809y4 - 23.92363 + 95.6076y2 - 162.5556y + 15.4792$$

– *Coefficient b*

Based on the coefficients presented in Table 19, we obtain:

Table 23. Tabulation of points (y, a).

y	7	6	5	4	3	2
a	-2	14,75	14,25	-20,5	-18,5	1,25

Source: Own (2022)

Based on the coefficients presented in Table 19, the following are obtained:

Table 24. Vector <y, a>.

y	b
7	-2
6	14,75
5	14,25
4	-20
3	-18,5
2	1,25

Following the procedure of the model presented, the interpolation polynomial of the coefficient b is calculated as shown:

Equation 14. Polynomial interpolation matrix of coefficient b.

A	B	C	D	E	F	
16807	2401	343	49	7	1	-2
7776	1296	216	36	6	2	14,75
3125	625	125	25	5	3	14,25
1024	256	64	16	4	4	-20,5
243	81	27	9	3	5	-18,5
32	16	8	4	2	6	1,25

Source: Own (2020)

Solution matrix or vector of coefficients of the interpolation function for the b coefficient.

A	1,4833333
B	-33,41667
C	285,58333
D	-1146,208
E	1956,8083
F	-187,5

Source: Own (2022)

Equation 15. Interpolation function of this coefficient.

$$b = 1{,}833y5 - 33{,}4167y4 + 285{,}5833y3 - 1146{,}2083y2 + 1956{,}8083y - 187{,}5$$

– *Coefficient c*

Based on the coefficients presented in Table 19, we obtain:

Table 25. tabulation of points (y, a).

y	7	6	5	4	3	2
c	0,8333333	1,75	-2,666667	-11,91667	-6,416667	-3,5

Source: Own (2022)

Table 26. Vector <y, a>.

y	c
7	0,8333333
6	1,75
5	-2,666667
4	-11,91667
3	-6,416667
2	-3,5

Source: Own 2022

Following the procedure of the proposed model, the interpolation polynomial of the coefficient c is calculated as shown:

Equation 16. Polynomial interpolation matrix of coefficient c.

A	B	C	D	E	F	
16807	2401	343	49	7	1	0,8333333
7776	1296	216	36	6	2	1,75
3125	625	125	25	5	3	-2,666667
1024	256	64	16	4	4	-11,91667
243	81	27	9	3	5	-6,416667
32	16	8	4	2	6	-3,5

Source: Own (2022)

Table 27. Tabulation of points (y, a).

A	0,4666667
B	-10,87153
C	96,756944
D	-407,1701
E	728,31806
F	-74,41667

Source: Own (2022)

Equation 17. Interpolation function of this coefficient.

$$c = 0.4667y5 - 10.8715y4 + 96.7569y3 - 407.1701y2 + 728.3181y - 74.4167$$

– *Coefficient d*

Based on the coefficients presented in Table 19, we obtain:

Table 28. Tabulation of points (y, a).

y	d
7	7
6	-46,5
5	-33
4	99,5
3	80,5
2	5

Source: Own (2020)

Following the procedure of the proposed model, the interpolation polynomial of the coefficient d is calculated as shown:

Equation 18. Polynomial interpolation matrix of the coefficient d.

A	B	C	D	E	F	
16807	2401	343	49	7	1	7
7776	1296	216	36	6	2	-46,5
3125	625	125	25	5	3	-33
1024	256	64	16	4	4	99,5
243	81	27	9	3	5	80,5
32	16	8	4	2	6	5

Source: Own 2022

Table 29. Solution matrix or vector of coefficients of the interpolation function for the d coefficient.

A	-5,733333
B	129,89583
C	-1117,708
D	4520,8542
E	-7773,308
F	752,5

Source: Own (2022)

Equation 19. Interpolation function of this coefficient.

$$d = -5.7333y5 + 129.8958y4 - 1117.7083y3 + 4520.8542y2 - 7773.3083y + 752.5$$

Finally it is summarized from the coefficients a, b, c, d, obtained as follows:

$$a = -0{,}125y^5 + 2{,}809y^4 - 23{,}9236y^3 + 95{,}6076y^2 - 162{,}5556y + 15{,}4792$$

$$b = 1{,}833y^5 - 33{,}4167y^4 + 285{,}5833y^3 - 1146{,}2083y^2 + 1956{,}8083y - 187{,}5$$

$$c = 0{,}4667y^5 - 10{,}8715y^4 + 96{,}7569y^3 - 407{,}1701y^2 + 728{,}3181y - 74{,}4167$$

$$d = -5{,}7333y^5 + 129{,}8958y^4 - 1117{,}7083y^3 + 4520{,}8542y^2 - 7773{,}3083y + 752{,}5$$

Surface Equation: Vehicles parked in R3 (Armas, W., (2019)

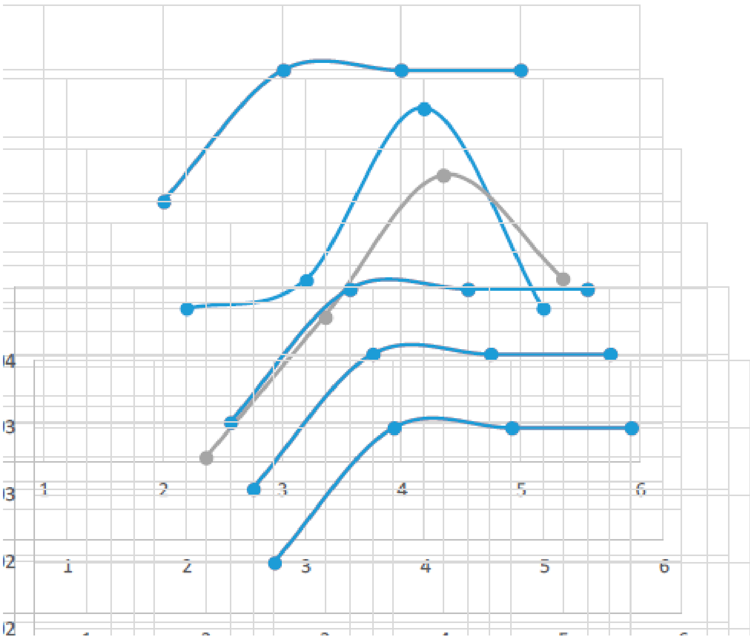
Applying Equation 11, it is obtained that:

Equation 20. Parked vehicles, in R3.

$$\begin{aligned} VP_{(x,y)} = {} & (-0{,}125y^5 + 2{,}809y^4 - 23{,}9236y^3 + 95{,}6076y^2 - 162{,}5556y + 15{,}4792)x^3 \\ & + (1{,}833y^5 - 33{,}4167y^4 + 285{,}5833y^3 - 1146{,}2083y^2 + 1956{,}8083y - 187{,}5)x^2 \\ & + (0{,}4667y^5 - 10{,}8715y^4 + 96{,}7569y^3 - 407{,}1701y^2 + 728{,}3181y - 74{,}4167)x \\ & + (-5{,}7333y^5 + 129{,}8958y^4 - 1117{,}7083y^3 + 4520{,}8542y^2 - 7773{,}3083y + 752{,}5) \end{aligned}$$

The modeling in R3 can be appreciated as the attainment of the graphs in R2 (Monographs (2019), along the y-axis, as shown:

Figure 2. attainment of the graphs in R2 along the y-axis.



Source: Own 2022

The modeling in R3 can be tabulated to simplify the programming of the calculations (Pérez-Vera, I. E. (2020), as follows:

Table 30. Table of coefficients of the surface equation.

Ci,j	y5	y4	y3	y2	y1	y0
x3	-0,125	2,809028	-23,92361	95,60764	-162,5556	15,47917
x2	1,483333	-33,41667	285,5833	-1146,208	1956,808	-187,5
x1	0,466667	-10,87153	96,75694	-407,1701	728,3181	-74,41667
x0	-5,733333	129,8958	-1117,708	4520,854	-7773,308	752,5

Calculation of maximum value points: strategic locations (Universidad politécnica de Madrid (2019). Applying Equation 15, we have:

Equation 21. System of equations for the solution of demand confluence points (Basurto Velásquez, J. C. (2022). Source: Own (2022)

$$\begin{cases} (-0{,}375y^5 + 8{,}427y^4 - 71{,}7708y^3 + 286{,}8229y^2 - 487{,}6667y + 46{,}4375)x^2 + \\ (2{,}9667y^5 - 66{,}8333y^4 + 571{,}1667y^3 - 2292{,}417y^2 + 3913{,}6167y - 375) \\ (0{,}4667y^5 - 10{,}8715y^4 + 96{,}7569y^3 - 407{,}1701y^2 + 728{,}3181y - 74{,}4167) = 0 \\ \\ (-0{,}625y^4 + 11{,}2361y^3 - 71{,}7708y^2 + 191{,}2153y - 162{,}5556)x^3 + (7{,}4167y^4 + \\ -133{,}6667y^3 + 856{,}75y^2 - 2292{,}417y + 1956{,}8083)x^2 + (2{,}3333y^4 - 43{,}4861y^3 \\ +290{,}2708y^2 - 814{,}3403y + 728{,}3181)x + (-28{,}6667y^4 + 519{,}5833y^3 - 3353{,}125y^2 \\ +9041{,}7083y - 7773{,}308) = 0 \end{cases}$$

From where (with the help of the Microsoft Excel Solver add-in) it is found that:

Table 31. Demand confluence points within the study area.

x (ver)	y (hor)	Dx	Dy	ΔD	VP(x,y)
3,56	4,51	0,20105	-0,096183	0,104867	29,27127
4,32	5,12	0,160143	-0,160218	-7,52E-05	35,44902
2,99	3,87	0,233528	-0,066505	0,167023	22,0082
4,07	2,15	0,404258	-0,152686	0,251572	7,847691

Source: Own (2022).

5. Conclusions

In this study, according to the application of the equations of (Cárdenas - Gutiérrez J.A, NJ Cely-Calixto, Carlos Acevedo Peñaloza, (2022) and applying the mathematical model, the points of confluence of the demand within the study area on the roads from 2nd Street to 7th Street are finally obtained with the help of Excel, as shown in Table 31 of this document.

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